River Flood Forecasting Using Complementary Muskingum Rating Equations

Parthasarathi Choudhury[1] and A. Sankarasubramanian[2]

Abstract: A model for real-time flood forecasting in river systems with large drainage areas has been developed. Flow variations between upstream and downstream stations are interlinked and are typically governed by reach properties. Unique paired variations establish useful flow correspondence resulting in inflow and outflow forecasting models for a reach. The proposed model can generate forecasts with increased lead time without applying a separate inflow forecasting model and can also provide updated forecasts essential for real-time applications. The model was applied to flood forecasting in Tar River Basin, N.C., covering a drainage area of 13,921 km^2. The model aggregates multiple upstream flows to provide long range forecasts for two downstream stations in the basin. Applicability of the model in estimating complete upstream and downstream hydrographs was demonstrated using a textbook example. Application results indicate that the new model can provide complete and updatable evolution of hydrographs using the current flow state.

DOI: 10.1061/(ASCE)HE.1943-5584.0000046

CE Database subject headings: Streamflow; Water demand; Forecasting; Surface water; Open channel flow; Drainage; North Carolina.

Introduction

Channel flooding is a complex dynamic process characterized by spatial and temporal variation in the flow parameters. Generally, information on water levels is collected at critical locations, as well as at existing stream gauging stations, for analyzing flood movement. Development of flood forecasting model characteristics based on only an observed stage is a difficult task because of recharge over the reach, spatial variability of rainfall, and varying channel characteristics influence river flow in a highly nonlinear manner. These issues become more complicated for large river systems, thus, requiring detailed distributed information for routing the flood along the river reach.

Deterministic flood forecasting models can be divided into two general categories: flood routing models and real-time rainfall–runoff models. Both types of models can be empirical, system-theoretic, or conceptual (Singh 1988). An operational forecasting system may, however, employ both models. One major disadvantage of most of the operational flood forecasting system is that the forecasts are often issued without a probabilistic statement on the forecast value. Toward this, many investigators have used filter theory (Georgakakos 1986; Puente and Bras1987; Bergman and Delleur 1985) to quantify the uncertainty in the forecasted flood values.

In river flood forecasting, applications of data driven models, such as artificial neural network (ANN), genetic algorithm, and fuzzy logic approaches can be found in the works of Thirumalaiah and Deo (1998), Chau et al. (2005), and Deka and Chandramouli (2005). ANN models are mostly used to predict downstream flows using multiple inputs namely past upstream and/or downstream flows rates in a reach. Francchini et al. (1994) developed a similar Muskingum model for predicting downstream flood levels. These models are useful for developing forecasts at a single location, a fixed time ahead with no provision for updating the forecasts. For real-time applications, forecasts updating capability is an essential requirement for a model. In the case of multiple inflows problem (Khan 1993; Choudhury 2007), similar difficulties exist with models lacking the ability in developing updated forecasts on a continuous basis.

In the case of a river reach, flow variation at the upstream end produces a specific variation at the downstream end and the variations are conjugate. The present study shows that two sets of unique conjugate flow variations representing complementary storage change for a reach can be defined. Unique paired flow variations lead to an upstream and downstream flow forecasting model for a reach. The model can be applied for updating forecasts in complex river systems with multiple tributaries. The methodology was employed for flood predictions in the Tar River Basin in North Carolina, experiencing severe hurricane related floods during the summer season. Applicability of the model in estimating complete upstream and downstream hydrographs is also demonstrated using a textbook example (Chow et al. 1988, Ex. 8.4.3, p. 269). Model performances are evaluated using standard statistical criteria. Results obtained show that the model is useful in real-time applications with an ability to update long range forecasts for multiple sections over a basin

Flood Forecasting Model

In a time interval Δt during a flood, flow at the upstream bounding section varies from an initial flow i_t to $i_{t+\Delta t}$ causing a variation

[1]Assistant Professor, Dept. of Civil Engineering, National Institute of Technology, Silchar, Assam, PIN 788010, India. E-mail: ps_chou@yahoo.com

[2]Assistant Professor, Dept. of Civil, Construction and Environmental Engineering, North Carolina State University, Raleigh, NC 27569-7908. E-mail: sankar_arumugam@ncsu.edu

Note. This manuscript was submitted on October 12, 2007; approved on October 31, 2008; published online on February 16, 2009. Discussion period open until December 1, 2009; separate discussions must be submitted for individual papers. This technical note is part of the ***Journal of Hydrologic Engineering***, Vol. 14, No. 7, July 1, 2009. ©ASCE, ISSN 1084-0699/2009/7-745–751/$25.00.

in the downstream flow and channel storage from q_t to $q_{t+\Delta t}$ and s_t to $s_{t+\Delta t}$, respectively, where i=inflow; q=outflow; s=storage; and subscripts t and $t+\Delta t$=current time and one time step in the future.

In the Muskingum routing model (Chow et al. 1988) flow variations at the bounding sections are expressed by a simple relationship as given in

$$q_{t+\Delta t} = c_1 i_t + c_2 i_{t+\Delta t} + c_3 q_t \tag{1}$$

$$s_t = k[x i_t + (1-x) q_t] \tag{2}$$

$$\frac{ds_t}{dt} = i_t - q_t \tag{3}$$

Here, the terms k denotes storage constant and x denotes the weight of inflow.

In the case of a river system draining a large area, routing can be accomplished by aggregating multiple inflows at a point in the basin (Choudhury 2007). Eq. (1) for a river system is written as

$$q_{t+\Delta t} = c_1 I_t^{e,r} + c_2 I_{t+\Delta t}^{e,r} + c_3 q_t \tag{4}$$

where

$$I_t^{e,r} = \sum_{p=1}^{n} \sigma^{p,r} i_t^p \tag{5}$$

Here, c_1, c_2, and c_3=routing coefficients; $I_t^{e,r}$=equivalent inflow at a point r in the basin for n flows measured at different locations; i_t^p=flow at a point p; $\sigma^{p,r}$=shift factor associated with the transfer of flow from p to r; and q_t=outflow at the common downstream station of the river system. It may be noted that flow i_t^p at p and $\sigma^{p,r} i_t^p$ at r are equivalent if the flows create the same effect at the downstream point and for a special Muskingum reach replacing a river system

$$I_t^{e,r} = \sum_{p=1}^{n} i_t^p = \sum_{p=1}^{n} \sigma^{p,r} i_t^p$$

The relationship in Eq. (1) expresses effects on flow and storage variation in a reach during time interval Δt. The model accounts for storage and flow evolution over time and can be rearranged to have present concurrent flows as inputs with future flows as outputs.

Eq. (1) shows that a river reach, having some wave propagation properties when subject to a flow variation at the upstream end and initial condition prevailing at the downstream section, determines outflow and channel storage evolution during the period. In other words, it may be stated that in a channel reach, outflow variation responds to inflow variation in a characteristic way governed by the properties of the channel. Further, the model given in Eq. (1) allows adjusting initial downstream flow commensurate with the upstream flow variation in a reach for creating a desired outflow response after a period Δt. By virtue of Eq. (1), a river reach where an upstream flow varying from i_t to $i_{t+\Delta t}$ causes outflow to vary from q_t to $q_{t+\Delta t}$ with storage changing by Δs, a flow variation at the upstream end from i_t' to $i_{t+\Delta t}$ with new initial flow q_t' at the downstream end can be selected that would produce no "end of the period" flow at the downstream section after a specified time interval Δt. The new set of initial conditions at the bounding sections and the upstream flow variation required to create a desired outflow response would be some function of the channel properties. Assuming that for a river reach with characteristics k and x an upstream flow varying from i_t' to $i_{t+\Delta t}$ with an initial flow given by $q_t = q_t'$ at the downstream side produce $q_{t+\Delta t}=0$ after a period Δt, then expressing the revised "start of the period" flow rates as a function of the respective observed flow at time t, Eqs. (2) and (3) may be used to write Eq. (1) as given in

$$c_1 \alpha i_t + c_3 \beta q_t = -(1 - c_1 - c_3) i_{t+\Delta t} \tag{6}$$

Here, α and β are the upstream flow evolution parameters. These parameters are functions of the upstream catchments, river reach, and the storm characteristics. The Muskingum routing equation given in Eq. (1) is useful for computing flow at time $(t+\Delta t)$ using known upstream/downstream flow at time $(t+\Delta t)$; the model cannot be used for issuing forecasts without using a separate flow forecasting model. It can be seen that by introducing parameters α and β and by eliminating c_2, the resulting equation given in Eq. (6) gives a predictive form of the model for estimating inflow at time step $(t+\Delta t)$.

Eq. (6) represents storage change in a river reach from s_t' to $s_t' + \Delta s'$ during the time interval Δt, where the initial state of the system is represented by altered storage s_t' and upstream and downstream flow rates as αi_t and βq_t, respectively. For a river reach, having parameters k and x, modified flow variation at the upstream and downstream end given by Eq. (6) are interlinked and conjugate similar to the observed paired variation given in Eq. (1). The new rating equation represents uncontrolled flood flow with revised initial conditions and obeys underlying physics of flood movement as given by the Muskingum model.

Combining Eqs. (6) and (1), the following equation may be obtained:

$$c_1(1-\alpha) i_t + c_3(1-\beta) q_t = q_{t+\Delta t} \tag{7}$$

Eqs. (6) and (7) represent inflow and outflow forecasting models for a river reach. From Eqs. (6) and (7), it can be seen that a value for α and β define both inflow and outflow trajectories and the equations are complementary depicting that for a prescribed evolution of upstream hydrograph, downstream hydrograph varies in a definite way.

Here, the conjugate flow variations from (1) αi_t to $i_{t+\Delta t}$ and βq_t to 0 and (2) $(1-\alpha) i_t$ to 0 and $(1-\beta) q_t$ to $q_{t+\Delta t}$ represent two unique sets of variations for the reach. The above-presented variations depict correspondence between inflow and outflow hydrographs and also represent that if $i_{t+\Delta t}$ corresponds to zero flow in the downstream side then flow αi_t must correspond to βq_t as a hydrograph pair. Variation in (1) depicts that at any time t a flow variation at the upstream side may be given producing a complete downstream hydrograph of duration $(t+\Delta t)$ and peak flow βq_t. Estimating upstream hydrograph evolution parameters α and β would facilitate application of Eqs. (6) and (7) for flood forecasting. It can be seen that estimates of α and β always satisfy continuity requirement of the simulation model given in Eq. (1). The main advantage of splitting the simulation model is that it gives inflow and outflow forecasting models, which may be used in conjunction to update forecasts for the bounding sections of a reach. The scope for updating forecasts is useful in real-time operation particularly in issuing flood warnings and in executing evacuation.

In case of a river system with a common outflow, complementary rating equations for Eq. (4) may be obtained by applying equivalent inflow as

$$c_1 \alpha I_t^{e,r} + c_3 \beta q_t = -(1 - c_1 - c_3) I_{t+\Delta t}^{e,r} \tag{8}$$

$$c_1(1-\alpha) I_t^{e,r} + c_3(1-\beta) q_t = q_{t+\Delta t} \tag{9}$$

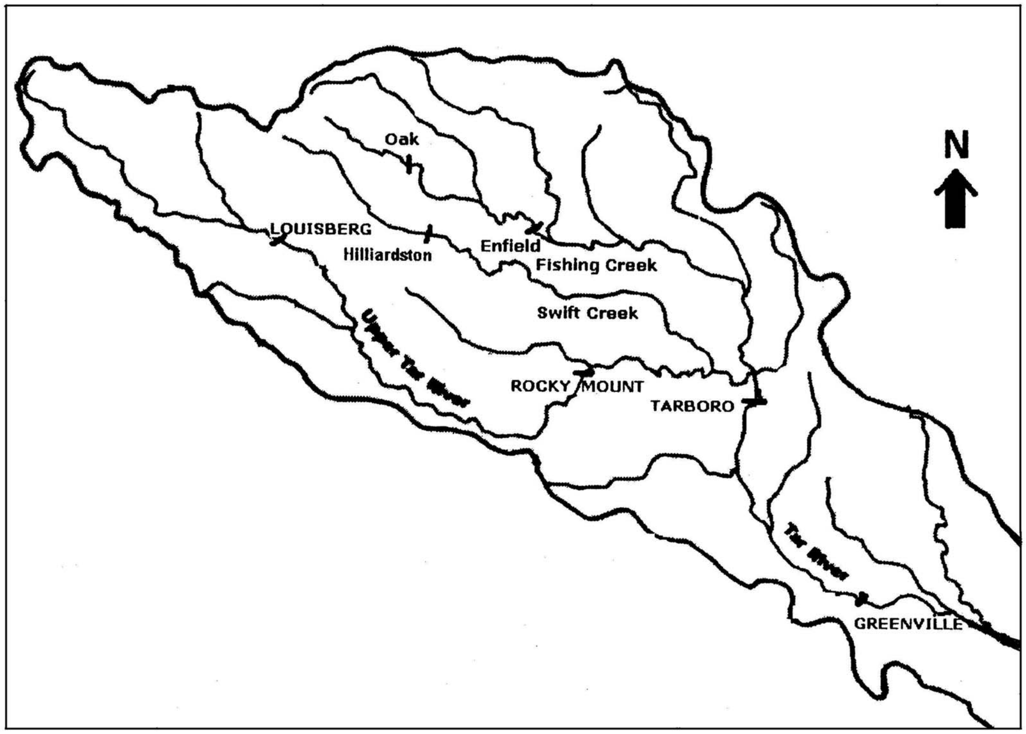

Fig. 1. Map of the study area in Tar River Basin

Eqs. (8) and (9) represent variations of equivalent inflow and the downstream outflow in an imaginary Muskingum reach replacing the network. Eqs. (8) and (9) may also be used for predicting equivalent inflow and the common downstream outflow in a river system. It is important to note that using the above-mentioned combination of single and multiple inflows model forms, flow forecasting in complex river systems may be accomplished.

Parameter Estimation

The new predictive model is applicable to both single and multiple reaches forming a river network. In the case of a single river reach using observed flow at time t, inflow and outflow at time $(t+1)$ and $(t+2)$ can be estimated by applying Eqs. (6) and (7). Comparing model estimated forecasts at time steps $(t+1)$ and $(t+2)$ with the observed flows, parameters c_1, c_3, α, and β, for a reach can be estimated by minimizing the following objective functions:

$$f(1) = \min \sum_{t=1}^{N-1} (i_{t+1} - \bar{i}_{t+1})^2 \quad (10)$$

$$f(2) = \min \sum_{t=1}^{N-1} (q_{t+1} - \bar{q}_{t+1})^2 \quad (11)$$

$$f(3) = \min \sum_{t=1}^{N-2} (i_{t+2} - \bar{i}_{t+2})^2 \quad (12)$$

$$f(4) = \min \sum_{t=1}^{N-2} (q_{t+2} - \bar{q}_{t+2})^2 \quad (13)$$

Where, $i_{(*)}$ and $q_{(*)}$=model generated predicted flow at the upstream and downstream sections, respectively; whereas $\bar{i}_{(*)}$ and $\bar{q}_{(*)}$ represent observed inflows and outflows, respectively. Minimization of the objective functions, Eqs. (10)–(13), lead to estimates for the model parameters c_1, c_3, α, and β. The predicted flows at time $(t+2)$ in Eqs. (12) and (13) are obtained using predicted flows at time $(t+1)$ as observed flow in Eqs. (6) and (7). The estimated parameters, when used to obtain forecasts for two future time steps, will show some forecast error due to estimation and measurement errors. Error at time $(t+2)$ may be expressed in terms of error at time $(t+1)$. To have an estimate of possible error in the next time step along with flow predictions, two additional objective functions as given in the following equations can be framed:

$$f(5) = \min \sum_{t=1}^{N-2} (E^i_{t+2} - \theta^i E^i_{t+1})^2 \quad (14)$$

$$f(6) = \min \sum_{t=1}^{N-2} (E^q_{t+2} - \theta^q E^q_{t+1})^2 \quad (15)$$

Optimal estimate of the parameters θ^i and θ^q describe a straight-line relationship between error at time $(t+1)$ and $(t+2)$ for the upstream and downstream stations, respectively. Here, $E^{(*)}_{(*)}$ represents error and $\theta^{(*)}$ denotes parameter that describes error propagation. Minimization of the six objective functions results in an estimate for the model parameters and the error parameters. With the parameter values for a river reach, the model may be used to

Table 1. Model Performance Statistics for Tar River System

	Tarboro				Greenville			
	2 hrs ahead		24 hrs ahead		2 hrs ahead		24 hrs ahead	
	RMSE (cfs)	DPF (cfs)	RMSE (cfs)	DPF (cfs)	RMSE (cfs)	DPF (cfs)	RMSE (cfs)	DPF (cfs)
Calibration	455.84	1,925.19	1,121.61	3,098.51	218.12	1,189.45	1,298.36	4,421.51
Verification	121.71	315.25	852.81	2,414.52	219.87	712.68	886.16	3,321.63
Parameters	$c_1=0.034$	$c_3=0.974$	$\alpha=0{,}237$	$\beta=-0.0001$	$c_1=0.226$	$c_3=0.810$	$\alpha=0.895$	$\beta=-0.208$
Error (θ)	Eq. In: 0.237		Tarboro: 0.587		Greenville: 0.543			
Shift ($\sigma^{p,r}$)	Rockymount: 0.395		Hilliardtson: 0.514		Enfield: 0.935			

Note: RMSE=root-mean-squared error and DPF=deviation in peak flow.

generate long range forecasts for the upstream and downstream stations. To obtain forecast with increased lead time, a procedure of making two consecutive forecasts using flow at time t, estimation of error at time $(t+2)$ on the basis of error at time $(t+1)$ and correction on the forecast at time $(t+2)$ could be incorporated.

Study Area: Tar River Basin

The proposed flood routing model formulation is demonstrated for the Tar-Pamlico River Basin in North Carolina. The Tar-Pamlico River Basin, with a drainage area of 13,921 km^2, is one of the four river basins that entirely lie within the state of North Carolina. The Tar River originates in north central North Carolina and flows southeasterly until it becomes the Pamlico River. Fig. 1 shows the map of the study area. Major tributaries of the Tar River main stem include Cokey Swamp, and Fishing, Swift and Sandy Creeks. In the Upper Tar River Basin, tributary flow from Swift Creek, Fishing and Little Fish Creeks contribute to the flow at Tarboro, N.C., and at Greenville, N.C.—a further downstream point in the river network. There is no significant tributary joining the river stretches from Tarboro to Greenville. This study uses present flow information available at Tarboro and its three upstream gauge sites, at Rocky Mount, Hilliardston, and Enfield to predict future flows at Tarboro and at a hypothetical site upstream of Tarboro. Using the flow information estimated for Tarboro, flow at Greenville is predicted. Concurrent flow records for these gauge sites starting from July 29, 2004 to October 01, 2004 with an interval of 15 min were collected from the U.S. Geological Survey (USGS) instantaneous streamflow data archive (http://ida.water.usgs.gov/ida/about_site.htm). Data sets with a 2-h interval were prepared from the original series and used for analyzing the flood events.

Applications

To apply the model for the Tar River Basin, the river network was replaced by an ordinary Muskingum reach. Equivalent flow for flows measured at the three upstream stations producing common outflow at Tarboro were used in Eqs. (8) and (9) to define a one step ahead forecasting model for the bounding section of the equivalent reach. To define forecasting models for the next reach from Tarboro to Greenville, single inflow–single outflow equations given in Eqs. (6) and (7) were used. To obtain model parameters c_1, c_3, α, and β, minimization of prediction error at time $(t+1)$ and $(t+2)$ for three sites were considered. Three additional objective functions were written for obtaining the best linear relationship between prediction error at time $(t+1)$ and $(t+2)$ for the sites. The objective functions were written as given in Eqs. (10)–(15). The objective functions were minimized using MatLab optimization function "lsqnonlin" (Mathworks Inc., Natick, Mass., version: 7.0.0.19920, R14). The function "lsqnonlin" can efficiently optimize multiple objective functions and is useful in nonlinear regression and in data fitting problems. In the present

Table 2. Results—Complete Upstream and Downstream Hydrograph Predictions (Chow et al. 1988, Ex. 8.4.3, p. 269)

Recorded flow (cfs)		Predicted flows (cfs)			
		Flow model		Error model	
Inflow	Outflow	Inflow	Outflow	Inflow	Outflow
60	0	60.00	0.00	60.00	0.00
180	42	256.66	55.96	319.84	116.88
300	127	375.55	182.21	434.67	240.89
446	231	486.24	303.23	540.40	358.76
613	363	587.07	417.17	635.50	468.71
776	514	676.60	522.31	718.61	569.11
932	672	753.59	617.12	788.63	658.50
932	822	817.03	700.25	844.66	735.65
911	879	866.14	770.58	886.04	799.53
941	897	900.38	827.20	912.38	849.36
975	924	919.48	869.45	923.52	884.58
980	954	923.40	896.90	919.56	904.88
951	968	912.34	909.39	900.83	910.23
890	956	886.77	906.98	867.91	900.78
810	919	847.35	889.98	821.59	876.98
717	851	794.98	858.93	762.84	839.46
618	769	730.73	814.58	692.86	789.08
514	677	655.86	757.90	612.96	726.87
410	579	571.76	690.01	524.63	654.07
309	478	479.96	612.23	429.43	572.04
248	373	382.08	525.97	329.03	482.26
229	302	279.81	432.79	225.14	386.33
216	260	174.88	334.32	119.50	285.90
205	235	69.03	232.24	13.84	182.69
$c_1=0.534$ $c_3=0.645$	RMSE (cfs)	94.23	78.19	85.19	69.91
$\alpha=0.434$ $\beta=-0.0833$	DPF (cfs)	58.60	57.96	58.58	57.77

Note: Alternate values of observed and predicted flows are listed. RMSE=root-mean-squared error and DPF=deviation in peak flow.

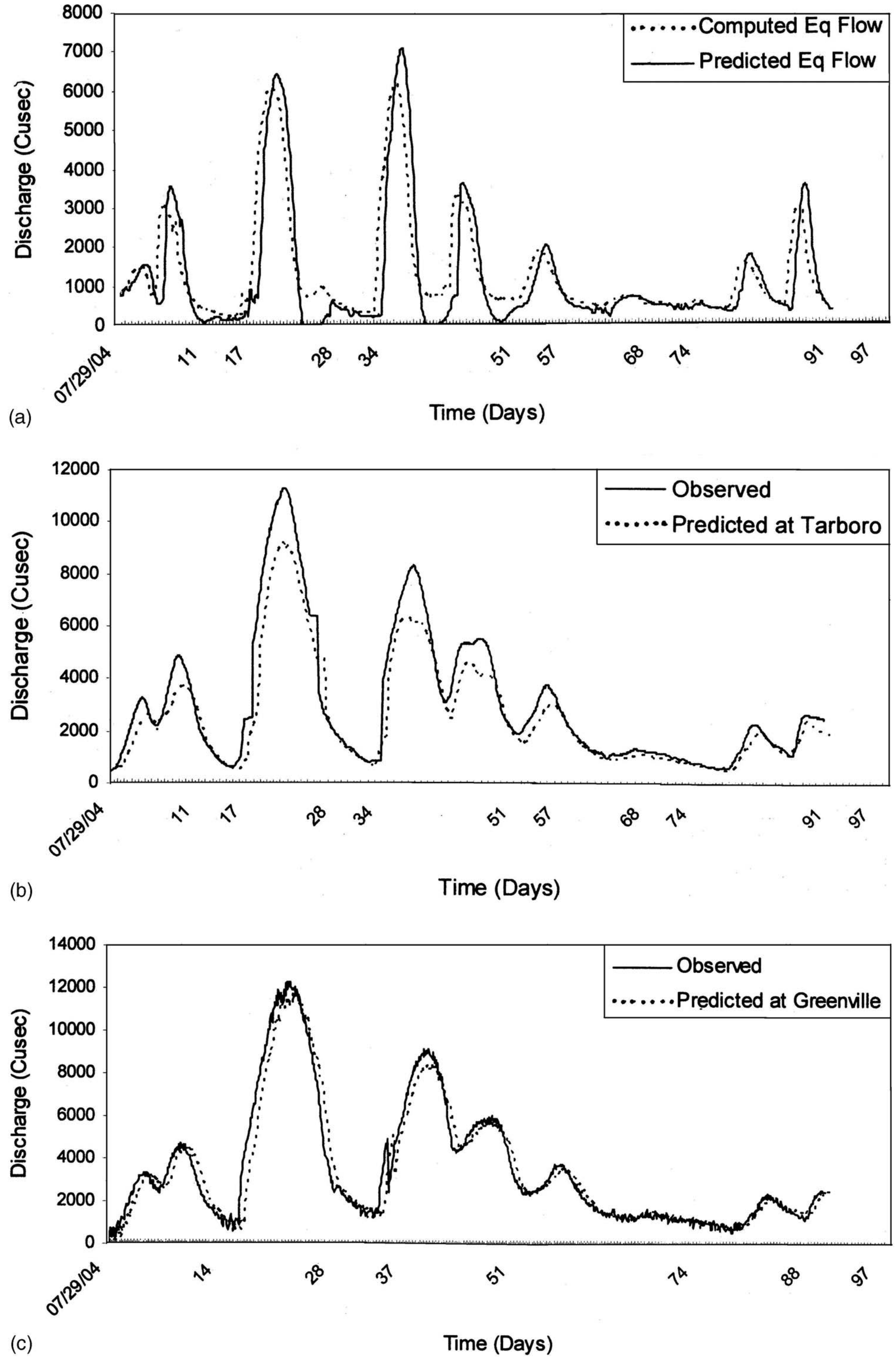

Fig. 2. (a) Computed and 24 h ahead predicted equivalent inflow; observed and 24 h ahead predicted flow at (b) Tarboro; and (c) GreenvilIe

study network, 14 parameters, including 3 error parameters necessary for forecast correction and 3 parameters necessary for defining the equivalent flow, were estimated minimizing 9 objective functions.

On the basis of equivalent inflow and flow at Tarboro at time t, the next equivalent flow and flow at Tarboro at time $(t+1)$ were predicted. The predicted flows at time $(t+1)$ were used to generate predictions at time $(t+2)$. To predict flow at Greenville at time $(t+2)$, predicted flow at Greenville at time $(t+1)$, and flow at Tarboro at time $(t+1)$ predicted on the basis of equivalent flow were used.

Nearly 1,100 concurrent flow measurements at five gauge sites at 2-h intervals were used in the study. First, 400 data sets covering two events were used to obtain model parameters and the remaining data were used to verify model performance. Estimates of the model parameters for the study network are listed in Table 1 along with performance statistics. The model parameters reflect characteristics of the routing reaches and are time-period dependent. The parameters α and β depict features of coincident flow variation in the upstream and downstream sections. The 24-h ahead forecast of equivalent inflow and forecast of downstream flow at Tarboro and Greenville obtained using the model are shown in Figs. 2(a–c). Results obtained show that the model can be used to develop long range (hourly to daily) forecasts for different stations in a complex river system. In real-time applications, as new observations are taken, the latest concurrent information can be used to update the model forecasts.

To illustrate further efficiency of the model in estimating entire inflow and outflow hydrographs, a text book example (Chow et al. 1988, Ex. 8.4.3, p. 269) is considered. The selected example represents a single inflow–single outflow problem with marginal (2.55%) ungauged lateral outflow from the reach. The problem was analyzed using Eqs. (6) and (7). Inflow hydrograph evolution parameters and reach parameters were estimated, minimizing four objective functions given in Eqs. (10)–(13). Parameter estimates are listed in Table 2. Using the estimated parameters in Eqs. (6) and (7), the entire inflow and outflow hydrographs were predicted from the observed initial conditions. As only initial observed values are used to predict hydrographs, prediction error accumulates with increasing prediction length. Predicted flows at time t show strong negative correlations with error at time $(t+1)$ indicating a nonlinear relationship, whereas the predicted series shows positive correlations with the observed flow at time $(t+1)$ indicating linear relationship. Taking linear functional form for both error and flow with predicted flows at previous time step, simple error and flow models as given in the following equation were developed:

$$P_{t+1} = \sigma_1 i_t + \sigma_2 q_t \tag{16}$$

where P_{t+1}=predicted flow /error at time $(t+1)$; i_t and q_t =inflow and outflow at time t obtained using initial observed flows; and $\sigma_{(*)}$=parameters that define distance of the actual flow/error trajectory at time $(t+1)$ with respect to the predicted flow plane at time t. The values of the parameters are listed in Table 3. Figs. 3(a and b) show hydrographs observed and predicted from the initial condition applying error and flow models. Observed and model generated predicted flows are listed in Table 2. Model performances were evaluated using root-mean-squared error (RMSE) and deviation in the peak flow (DPF) criteria (Yoon and Padmanabhan 1993). The results in Table 2 show that both error and flow models perform similarly. It can be found from the results in Table 2 and Figs. 3(a and b) that the model yields equally good results in estimating hydrographs, matching closely with the observed hydrographs. Results shown from modeling Tar River flows and from the standard textbook problem indicate that using the current flow state the model can provide complete and updatable evolution of hydrographs in a river reach.

Table 3. Observed Flow and Error Model Parameters

	Parameters			
	Flow model		Error model	
Prediction	σ_1	σ_2	σ_1	σ_2
Inflow	4.278	−0.736	−3.039	0.487
Outflow	0.933	2.622	−0.584	−1.969

Conclusions

Flood forecasting in multiple sections of a river basin could be accomplished using complementary rating equations. Flow variation at the bounding sections was split into two unique sets of conjugate variations leading to complementary Muskingum rating equations. The new variations establish useful correspondence between inflow and outflow hydrographs leading to the development of inflow and outflow forecasting models. A combination of single and multi-inflows model form is useful for developing a flood forecasting model in complex river systems. The model formulation was applied for flood forecasting in Tar River Basin, N.C. The network portion of the river system was replaced by an ordinary Muskingum reach with an equivalent inflow. The model was used to predict equivalent inflow and two downstream flows in the system. Efficiency of the model in obtaining long range updatable forecasts was further demonstrated using a textbook example. Complete inflow and outflow hydrographs for the selected event were estimated from the observed initial state.

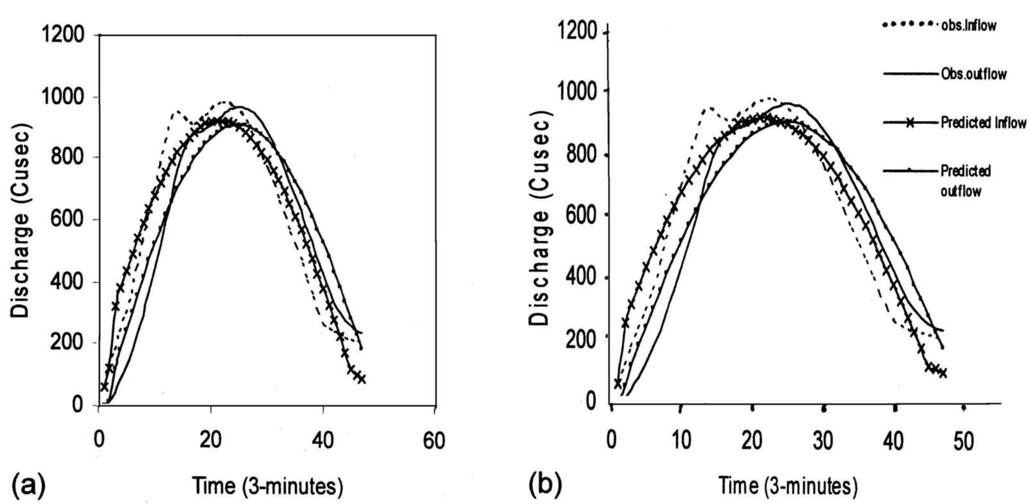

Fig. 3. Complete flood event prediction (Chow et al. 1988): (a) error model: predicting the error using predicted flows; (b) flow model: predicting the observed flows using predicted flow at the bounding sections obtained from initial state

Model performance analysis shows that the model is efficient in developing and updating hourly to daily forecasts. Given the model's ability to update forecasts, the proposed formulation has utility in real-time applications particularly in issuing flood warnings.

Notation

The following symbols are used in this technical note:

c_1, c_2, c_3 = Muskingum routing coefficients;

$E_{(*)}^{(*)}$ = error;

$I_t^{e,r}$ = equivalent inflow at a point r in the basin;

i_t = model generated inflow at time t;

t_t' = revised inflow at time t;

k = Muskingum model parameter having dimension of time;

q_t = model generated outflow at time t;

q_t' = revised outflow at time t;

$s_t; s_t'$ = initial storage at time t;

x = a weighting factor in the Muskingum model;

α, β = upstream hydrograph evolution parameters;

Δt = time period;

Δs = storage during time period Δt;

$\Delta s'$ = complementary storage change during time period Δt;

$\theta_{(*)}^{(*)}$ = parameter for the best fitted straight line;

$\sigma^{p,r}$ = shift factor associated for transfer of flow from p to r; and

$\sigma_{(*)}$ = parameters defining actual flow/error trajectory.

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